

Gauge Fields and Unparticles

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Abstract

We show that a rigorous path integral method of introducing gauge fields in the UnParticle lagrangian leads to somewhat different and more complicated vertexes than those currently used.

INTRODUCTION

The idea of a scalar field that represents a particle of indefinite mass, introduced by Georgi [1] [2], was extended to a gauged field by Terning et al [3]. The unparticle action was taken in the nonlocal form:

$$I = \int d^4x d^4y \bar{\psi}(x) K(x-y) \psi(y) \quad (1)$$

Where K denotes the inverse of the Unparticle's propagator. To include a gauge field A , an additional term is included:

$$U(x, y, \gamma) = P \left[\exp \left(-ig \int_{x, \gamma}^y A_\mu(w) dw^\mu \right) \right] \quad (2)$$

Here P denotes a path ordering, and γ denotes a path from x to y . Then

$$I = \int d^4x d^4y \bar{\psi}(x) K(x-y) U(x, y, \gamma) \psi(y) \quad (3)$$

Terning et al [3] do not specify the path γ , but they make the assumption that it is always such that

$$\frac{\partial}{\partial y^\mu} U(x, y, \gamma) = -ig U(x, y, \gamma) A_\mu(y) \quad (4)$$

This is a very old idea that goes back to Mandelstam [4], but it can not be quite correct. It requires that for all x and y , the path that goes from x to $y + dy$ must have gone first from x to y . However, the paths between all point pairs must be exactly defined before the integral in Eq. (3) can be calculated. Whatever the definition of the path from x to $y + dy$, it cannot be expected to have gone through y just because someone wishes to compute the derivative.

In the following, we investigate the consequences of defining the path integral between any two points x and y as the straight line from x to y . We show that the resulting vertexes satisfy the Ward-Takahashi identities [5], but they do lead to vertexes that are rather more complicated than those found in Ref. [3]. In a later work, we will show that a Terning-type vertex can be obtained by a different method of introducing gauge fields into the Unparticle action.

STRAIGHT LINE PATH

Choosing the path as the straight line from x to y leads to

$$U(x, y) = P \left[\exp \left(-ig \int_0^1 A_\mu(w(\lambda)) dw^\mu(\lambda) \right) \right] \quad (5)$$

where

$$w^\mu(\lambda) = (1 - \lambda)x^\mu + \lambda y^\mu \quad (6)$$

The UP-gauge-UP vertex is defined by

$$ig\Gamma^\mu(y, x, z) = - \frac{\delta^3 I}{\delta A_\mu(x) \delta \psi(y) \delta \bar{\psi}(z)} \Big|_{A=0} \quad (7)$$

Fourier transforming:

$$ig\Gamma^\mu(p, q, p+q) (2\pi)^4 \delta(p' - p - q) = \int d^4x d^4y d^4z e^{i(p'z - py - qx)} ig\Gamma^\mu(y, x, z) \quad (8)$$

Using

$$\frac{\delta}{\delta A_\mu(w)} U(x, y) \Big|_{A=0} = -ig \int_0^1 d\lambda \delta(w - (1 - \lambda)x - \lambda y) (y^\mu - x^\mu) \quad (9)$$

and with $S(k)$ denoting the Unparticle propagator in momentum space,

$$K(x - y) = \int \frac{d^4k}{(2\pi)^4} S^{-1}(k) e^{ik(x-y)} \quad (10)$$

we get

$$\Gamma^\mu(p, q, p+q) = -i \int_0^1 d\lambda \frac{\partial}{\partial k_\mu} S^{-1}(k) \Big|_{k=-(p+\lambda q)} \quad (11)$$

We show that this satisfies the Ward-Takahashi identity. We consider first the scalar UP case, where the propagator depends on k through $s = k^2 = p^2 + 2(p \cdot q)\lambda + q^2\lambda^2$. Then

$$\Gamma^\mu = 2i \int_0^1 d\lambda (p^\mu + \lambda q^\mu) \frac{dS^{-1}}{ds} \quad (12)$$

and with $\frac{ds}{d\lambda} = 2(p \cdot q + \lambda q^2)$ we get

$$q^\mu \Gamma_\mu = i \int_0^1 d\lambda \frac{ds}{d\lambda} \frac{dS^{-1}}{ds} = i [S^{-1}(p+q) - S^{-1}(p)] \quad (13)$$

the WT relation. If now the UP is a fermion, then

$$S^{-1} = \gamma^\mu k_\mu g(s) \quad (14)$$

and

$$\Gamma^\mu = -i \int_0^1 d\lambda \left[\gamma^\mu g + 2\gamma^\alpha (p_\alpha + \lambda q_\alpha) (p^\mu + \lambda q^\mu) \frac{dg}{ds} \right] \quad (15)$$

then

$$\begin{aligned} q^\mu \Gamma_\mu &= -i \int_0^1 d\lambda \gamma^\alpha \left[q_\alpha g + (p_\alpha + \lambda q_\alpha) \frac{dg}{d\lambda} \right] \\ &= i\gamma^\alpha \left[(p_\alpha + q_\alpha) g((p+q)^2) - p_\alpha g(p^2) \right] \\ &= i \left[S^{-1}(p+q) - S^{-1}(p) \right] \end{aligned}$$

the WT identity.

THE VERTEX INTEGRAL

It is generally assumed [1] [2] that the Fourier transform of the inverse propagator goes as a power of the invariant momentum squared:

$$S^{-1}(k) \doteq (k^2)^\nu \quad (16)$$

where $\nu = 2 - d_u$, d_u being the unparticle dimension. We therefore need to find the integral

$$f_\nu(p, p') = \int_0^1 s^\nu d\lambda \quad (17)$$

This can be written as

$$f_\nu = \frac{1}{2\sqrt{q^2}} \int_{s_0}^{s_1} ds \frac{s^\nu}{\sqrt{s-A}} \quad (18)$$

where $s_0 = p^2$, $s_1 = p'^2$ and

$$A = p^2 - \frac{(p \cdot q)^2}{q^2} \quad (19)$$

If ν is not a half integer, the integral in Eq. (18) can be done as an infinite series, giving

$$f_\nu(p, p') = g_\nu(p'^2, A) - g_\nu(p^2, A) \quad (20)$$

where

$$g_\nu(s, A) = \frac{s^{\nu+\frac{1}{2}}}{2\sqrt{q^2}} \sum_{k=0}^{\infty} \frac{(2k-1)!!}{2^k k! (\nu - k + \frac{1}{2})} \left(\frac{A}{s}\right)^k \quad (21)$$

THE SCALAR VERTEX

In this section we show another way of calculating the vertex integral and derive the vertex for a scalar unparticle. The vertex is given in terms of the vertex integral as

$$\Gamma^\mu = 2i \left. \frac{\partial}{\partial p^\mu} f_\nu \right|_q \quad (22)$$

where the scalar integral is

$$f_\nu = \int_0^1 d\lambda (s(\lambda))^\nu \quad (23)$$

with $s = (p + \lambda q)^2$, this can be expanded in a Taylor series in λ and then integrated to give

$$f_\nu = A^\nu \sum_{k=0}^{\infty} \frac{\Gamma(\nu+1)}{k! (2k+1) \Gamma(\nu-k+1)} \left(\frac{q^2}{A}\right)^k \left[(1+B)^{2k+1} - B^{2k+1}\right] \quad (24)$$

where

$$\begin{aligned} A &= p^2 - \frac{(p \cdot q)^2}{q^2} \\ &= \frac{p^2 p'^2 - (p \cdot p')^2}{(p' - p)^2} \end{aligned}$$

and

$$\begin{aligned} 1+B &= \frac{p' \cdot q}{q^2} \\ B &= \frac{p \cdot q}{q^2} \end{aligned}$$

This can be expressed in terms of the hypergeometric functions

$${}_2F_1(\alpha, \beta, \gamma; z) = 1 + \frac{\alpha\beta}{\gamma} z + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\gamma(\gamma+1)2!} z^2 + \dots \quad (25)$$

as

$$f_\nu = A^\nu \left[(1+B) {}_2F_1\left(\frac{1}{2}, -\nu, \frac{3}{2}; -\frac{q^2}{A}(1+B)^2\right) - B {}_2F_1\left(\frac{1}{2}, -\nu, \frac{3}{2}; -\frac{q^2}{A}B^2\right) \right] \quad (26)$$

Defining

$$Q_\nu(z) = {}_2F_1\left(\frac{1}{2}, -\nu, \frac{3}{2}; -\frac{q^2}{A}z^2\right) \quad (27)$$

and using

$$\left. \frac{\partial B}{\partial p_\mu} \right|_q = \frac{q^\mu}{q^2} \quad (28)$$

$$\begin{aligned} \left. \frac{\partial A}{\partial p_\mu} \right|_q &= 2 \left(p^\mu - \frac{(p \cdot q)}{q^2} q^\mu \right) \\ &= \frac{2}{q^2} [p^\mu (p' \cdot q) - p'^\mu (p \cdot q)] \\ C_\nu &= (1 + B) Q_\nu (1 + B) - B Q_\nu (B) \end{aligned} \quad (29)$$

we get

$$\Gamma^\mu = 2i \left\{ \frac{q^\mu}{q^2} [p'^{2\nu} - p^{2\nu}] + \frac{2\nu A^{\nu-1}}{q^2} [p^\mu (p' \cdot q) - p'^\mu (p \cdot q)] C_{\nu-1} \right\} \quad (30)$$

This is considerably more complicated than the result found in Ref. [3].

When $\nu = 1$, this reduces to

$$\Gamma^\mu = 2i (p'^\mu + p^\mu)$$

The expected result for a scalar particle.

CONCLUSIONS

We find that a rigorous application of the path integral method of introducing gauge fields into the unparticle Lagrangian leads to vertexes that are considerably more complicated than those found by Terning et al. [3] We will show in a later work that there is another method of combining gauge fields and unparticles does lead to the Terning result.

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